

# Broken $\mu$ - $\tau$ Symmetry and Leptonic $CP$ Violation

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## Abstract

We propose that leptonic  $CP$  violation arises from a breaking of the  $\mu$ - $\tau$  exchange symmetry in the neutrino mass matrix which in turn is indicated by the near maximal atmospheric neutrino mixing and the near zero  $\theta_{13}$ . We find that for the case of a normal hierarchy, present data already restricts the way  $CP$  violation may appear in the neutrino mass matrix and there is an interesting correlation between the mixing angle  $\theta_{13}$ , the solar mixing angle  $\theta_{12}$  and the Dirac  $CP$  phase. In the inverted hierarchy only  $\theta_{13}$  and the Dirac phase are linked. We also discuss the impact of this kind of  $CP$  violation on the deviation of the atmospheric mixing from its maximal value. Moreover, if corrections to  $\mu$ - $\tau$  symmetry arise from the charged lepton sector, where  $\mu$ - $\tau$  symmetry is known to be broken anyway, we find interesting connections between the  $CP$ -even and -odd terms as well. Our predictions are testable in the proposed long baseline neutrino experiments.

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# 1 Introduction

Recent discoveries in neutrino physics have opened up an interesting window into physics beyond the standard model. Aside from the very fact that the neutrinos have mass, there are two surprising sets of results (not in line with expectations based on our knowledge of the quark sector) that have provided new opportunities for the field: they are (i) the extreme smallness of the overall value of the neutrino masses and (ii) the dramatically different mixing pattern from quarks. The first one can be interpreted as an indication that perhaps a new symmetry such as  $B - L$  may be appearing at high scales so that one can use the seesaw mechanism using the right-handed neutrinos to resolve the puzzle of small masses. The mixing pattern on the other hand poses a completely different and much more challenging problem. It is expected that solving this problem will reveal new symmetries for leptons and perhaps as well for quarks.

A fundamental theory can of course determine the structure of both the charged lepton and the neutrino mass matrices and therefore will lead to predictions about lepton mixings. However, in the absence of such a theory, one wants to adopt a model independent approach and look for symmetries that may explain the mixing pattern which in turn can throw light on the nature of the fundamental theory for quarks and leptons.

One such symmetry has been inspired by the near maximal value of  $\theta_{23}$  and the near zero value of  $\theta_{13}$ . These angles appear in the usual definition of the PMNS matrix

$$U = U_\ell^\dagger U_\nu = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\ -c_{23} s_{12} - s_{23} s_{13} c_{12} e^{i\delta} & c_{23} c_{12} - s_{23} s_{13} s_{12} e^{i\delta} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} s_{13} c_{12} e^{i\delta} & -s_{23} c_{12} - c_{23} s_{13} s_{12} e^{i\delta} & c_{23} c_{13} \end{pmatrix}, \quad (1)$$

where  $U_\nu$  diagonalizes the neutrino mass matrix via  $U_\nu^T \mathcal{M}_\nu U = \mathcal{M}_\nu^{\text{diag}}$  and  $U_\ell$  is associated with the diagonalization of the charged leptons. If one works in a basis where charged leptons are mass eigenstates, then  $U_\ell^\dagger = \mathbb{1}$  and a simple way to understand  $\theta_{23} = \pi/4$  and  $\theta_{13} = 0$  is to postulate that the neutrinos obey a  $\mu$ - $\tau$  interchange symmetry [1, 2, 3, 4, 5]. Even though this procedure is a basis dependent one, the hope is that any symmetries for leptons revealed in this basis are true or approximate symmetries of Nature itself<sup>1</sup>.

A convenient parameterization of the neutrino mass matrix in this basis that obeys exact  $\mu$ - $\tau$  symmetry is given for the case of normal hierarchy by [5]:

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} d \epsilon^n & a \epsilon & a \epsilon \\ a \epsilon & 1 + \epsilon & -1 \\ a \epsilon & -1 & 1 + \epsilon \end{pmatrix}, \quad (2)$$

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<sup>1</sup>One might think that since the muon and tau lepton masses are so different, such a symmetry cannot be present in Nature. While this could be true, it is worth remembering that in the context of supersymmetric models, the origin of the up-type fermion masses (i.e.,  $u, c, t$  quark masses and the neutrino masses) and those of the down type fermion masses ( $d, s, b$  quarks and  $e, \mu$  and  $\tau$  leptons) arise from different Higgs doublets, the so-called  $H_u$  and  $H_d$  fields. Therefore, one could easily imagine a situation where  $H_u$  and the quarks are all singlets under the  $\mu$ - $\tau$  exchange symmetry whereas  $H_d$ , the leptons and the right-handed neutrinos are not. In a situation like this, the neutrinos could respect  $\mu$ - $\tau$  symmetry whereas the charged leptons need not.

where  $n \geq 1$  and  $\epsilon \ll 1$ . An immediate prediction of this mass matrix is that  $\theta_{23} = \pi/4$ ,  $\theta_{13} = 0$  and furthermore  $\epsilon \sim \sqrt{\Delta m_{\odot}^2 / \Delta m_A^2}$ . Note that this matrix conserves  $CP$  symmetry. This suggests several ways for introducing  $CP$  violation in the leptonic sector.

$CP$  violation could either be introduced while maintaining  $\mu$ - $\tau$  symmetry or by breaking it. In the first case, the most general form is

$$\mathcal{M}_\nu = \frac{\sqrt{\Delta m_A^2}}{2} \begin{pmatrix} d\epsilon^n & a\epsilon e^{i\alpha} & a\epsilon e^{i\alpha} \\ a\epsilon e^{i\alpha} & 1+\epsilon & -e^{i\gamma} \\ a\epsilon e^{i\alpha} & -e^{i\gamma} & 1+\epsilon \end{pmatrix}. \quad (3)$$

While this matrix still leads to maximal atmospheric neutrino mixing and vanishing  $U_{e3}$ , it destroys another nice feature, i.e., the smallness of  $\sqrt{\Delta m_{\odot}^2 / \Delta m_A^2}$ , which is not any more directly connected to the small parameter  $\epsilon$ . In fact, the small solar mass squared difference and the large solar neutrino mixing require that  $\gamma$  be very small, i.e.,  $\gamma \leq \sqrt{\Delta m_{\odot}^2 / \Delta m_A^2}$ , so that the 2–3 sub-determinant of the mass matrix is small. Therefore, in what follows, we set  $\gamma = 0$ . This case is essentially the same as the  $CP$  conserving case since the new phase simply changes the Majorana phases in the PMNS matrix and does not manifest itself in oscillation phenomena. The same conclusion holds if a phase is put in the  $ee$  entry.

On the other hand, if  $CP$  violation arises because of  $\mu$ - $\tau$  symmetry breaking, a priori one could imagine putting the  $CP$  phase in different places in the matrix. If we put the phase in the  $\mu\mu$  or  $\tau\tau$  entry, then it has to be close to zero in order to make the lower 2–3 sub-determinant small. Hence, the only position where a phase breaking the  $\mu$ - $\tau$  symmetry could be put is either the  $e\mu$  entry or the  $e\tau$  entry. Both are equivalent and we therefore discuss the case where only the  $e\tau$  entry of the neutrino mass matrix Eq. (2) is complex [6].

The situation for the case of inverted hierarchy is somewhat more complicated and is discussed later on in the text.

In this paper, we pursue the consequences of this hypothesis for both the normal and inverted hierarchy and show that it has several interesting consequences, linking for instance the size of  $\theta_{13}$  and  $\theta_{23} - \pi/4$  with the magnitude of leptonic  $CP$  violation and the value of the solar neutrino mixing angle. These correlations depend on whether the neutrino mass hierarchy is normal or inverted.

A different way to break  $\mu$ - $\tau$  symmetry is when contributions from the charged lepton sector perturb the PMNS matrix. One could imagine that the neutrino mass matrix is completely  $\mu$ - $\tau$  symmetric and corrections stem entirely from the charged lepton sector, which is known to break  $\mu$ - $\tau$  symmetry anyway. We also analyze this case and show there can also be interesting correlations between the observables. Unlike the previous case, the results for this case are independent of the mass hierarchy of the neutrinos.

## 2 Normal hierarchy

In this section, we discuss the case when the charged lepton mass matrix is diagonal and a leptonic  $CP$  violating phase is present in the neutrino mass matrix in a way that it breaks

$\mu$ - $\tau$  symmetry. This leads to very interesting phenomenology. Consider the following matrix<sup>2</sup>:

$$\mathcal{M}_\nu = \frac{m_0}{2} \begin{pmatrix} d\epsilon^2 & a\epsilon & a\epsilon e^{i\alpha} \\ a\epsilon & 1+\epsilon & -1 \\ a\epsilon e^{i\alpha} & -1 & 1+\epsilon \end{pmatrix}, \quad (4)$$

where  $\epsilon \ll 1$  and  $a, d$  are real parameters of order one<sup>3</sup>. It is clear from the discussion above that in the limit of  $\alpha = 0$  exact  $\mu$ - $\tau$  symmetry is recovered and the values  $\theta_{23} = \pi/4$  and  $U_{e3} = 0$  are predicted. It is straightforward to obtain the phenomenology of this matrix: first, the effective mass governing neutrinoless double beta decay is very much suppressed (for a recent overview of the situation regarding the effective mass, see, e.g., [7]). Turning to  $CP$  violation in neutrino oscillations, all such observables are described in terms of one rephasing invariant, namely [8]:

$$J_{CP} = \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta = \frac{\text{Im} \{h_{12} h_{23} h_{31}\}}{\Delta m_{21}^2 \Delta m_{31}^2 \Delta m_{32}^2}, \quad (5)$$

where  $h = \mathcal{M}_\nu^\dagger \mathcal{M}_\nu$ .

For the matrix in Eq. (4) we have that

$$\text{Im} \{h_{12} h_{23} h_{31}\} \simeq -\frac{m_0^6}{16} a \epsilon^3 \sin \alpha + \mathcal{O}(\epsilon^4). \quad (6)$$

As it should, this expression (as well as the more lengthy exact one) vanishes for  $\alpha = 0$ . Of course, one cannot identify  $\delta$  in the parametrization of the PMNS matrix (1) with  $\alpha$ , even though  $J_{CP}$  is proportional to  $\sin \alpha$ . This can be understood as follows: in our model,

$$|U_{e3}| \simeq \frac{a}{2} \epsilon \sqrt{1 - \cos \alpha} \quad \text{and} \quad \sin \delta \simeq -\cos \frac{\alpha}{2}. \quad (7)$$

As a result,  $\delta$  formally does not vanish for  $\alpha = 0$ . There is no inconsistency in this result, because the invariant  $J_{CP}$  vanishes if  $\alpha = 0$ . We see from Eq. (7) that  $\alpha = 0$  corresponds to  $U_{e3} = 0$ , which leaves the Dirac phase undefined. What is interesting is that there is an inverse correlation between  $U_{e3}$  and the Dirac phase  $\delta$ .

Solar neutrino mixing is governed by

$$\tan 2\theta_{12} \simeq 2a \sqrt{1 + \cos \alpha}, \quad (8)$$

which for  $\alpha = 0$  agrees with the result from [5]. The value of  $\alpha = \pi$ , which would mean maximal  $U_{e3}$  and no  $CP$  violation, is not allowed because it would result in  $\theta_{12} = 0$ . In

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<sup>2</sup>Breaking the symmetry by a complex entry in the  $e\mu$  instead of the  $e\tau$  element will give basically the same results, with the only change that  $\alpha \rightarrow -\alpha$ .

<sup>3</sup>This particular form of the mass matrix is motivated by approximate leptonic symmetries of the type  $U(1)_{L_e}$  times the  $\mu$ - $\tau$  exchange symmetry, where  $\epsilon$  denotes the strength of  $U(1)_{L_e}$  breaking. This can keep the  $\mu$ - $\tau$  sector to have entries of order one, the  $e\mu$  and  $e\tau$  elements of order of  $\epsilon$  and the  $ee$  entry of order  $\epsilon^2$ .

Figure 1 we show for a particular choice of the parameters  $a, b, \epsilon$ , the behavior of  $|U_{e3}|$ ,  $\sin \delta$  and  $\sin^2 \theta_{12}$  as a function of  $\alpha$ . Though maximal  $CP$  violation (i.e.,  $\sin \delta = \pm 1$ ) occurs only when  $U_{e3} = 0$ , and maximal  $|U_{e3}|$  occurs only when  $J_{CP} = 0$ , we can have large values of  $\sin \delta$  for sizable and testable values of  $|U_{e3}|$ : for instance  $\sin \delta \simeq 0.5$  with  $|U_{e3}| \simeq 0.05$ . Instead of scanning the whole parameters space of the model, we note here that the following ratio of observables depends only on the  $\mu$ - $\tau$  symmetry breaking parameters and not on the order one parameters  $a$  or  $d$ :

$$\frac{|U_{e3}|}{\tan 2\theta_{12}} \simeq \frac{\epsilon}{4} \tan \frac{\alpha}{2} \simeq \frac{\epsilon}{4} \cot \delta . \quad (9)$$

Atmospheric neutrino mixing stays very close to maximal and is of course maximal for a vanishing phase:

$$\theta_{23} - \frac{\pi}{4} \simeq -\frac{a}{2} \epsilon^2 \sin \alpha / 2 . \quad (10)$$

Hence, the deviation from  $\sin^2 \theta_{23} = 1/2$  is of order  $\epsilon^2$ , so that next generation experiments [9] will not be able to measure any non-maximality. The required precision can only be achieved by far-future projects like neutrino factories or  $\beta$ -beams.

Finally, the mass squared differences are given by

$$\begin{aligned} \Delta m_A^2 &\simeq m_0^2 (1 + \epsilon) , \\ \Delta m_\odot^2 &\simeq \frac{m_0^2}{4} \sqrt{1 + 4a^2 (1 + \cos \alpha)} \epsilon^2 , \end{aligned} \quad (11)$$

where we set  $d\epsilon^2$  to zero to keep tractable expressions. We see that as in the case of  $\mu$ - $\tau$  breaking with real parameters, the ratio of solar and atmospheric  $\Delta m^2$  is of order  $\epsilon^2$ , which is also the order of  $|U_{e3}|^2$  [5]. The presence of a non-trivial phase can however slightly spoil this simple behavior. The results for the oscillation parameters were obtained by the usual perturbative method, in which first the 23-block of Eq. (4) is diagonalized, then the 13- and after that the 12-block. Details of this standard procedure can be found, e.g., in Ref. [11].

A different  $\mu$ - $\tau$  breaking is possible when the  $e\mu$  and  $e\tau$  entries are complex conjugates [3]:

$$\mathcal{M}_\nu = \frac{m_0}{2} \begin{pmatrix} d\epsilon^2 & a\epsilon e^{-i\alpha} & a\epsilon e^{i\alpha} \\ a\epsilon e^{-i\alpha} & 1 + \epsilon & -1 \\ a\epsilon e^{i\alpha} & -1 & 1 + \epsilon \end{pmatrix} . \quad (12)$$

The predictions of this matrix are  $\theta_{23} - \frac{\pi}{4} \simeq a^2 \epsilon^2 \cos \alpha \sin \alpha$ , maximal  $CP$  violation (i.e.,  $\delta = \pm\pi/2$ ) and  $|U_{e3}| \simeq a/\sqrt{2} \sin \alpha \epsilon$ . Moreover, we find that  $\tan 2\theta_{12} \simeq 2\sqrt{2}a \cos \alpha$  and  $\Delta m_\odot^2 \simeq m_0^2/4 \sqrt{1 + 8a^2 \cos^2 \alpha}$ . Though the phase  $\alpha$  does not affect the size of the low energy phase  $\delta$ , it is again crucial for the magnitude of  $|U_{e3}|$  and for obtaining large solar neutrino mixing: a maximal  $U_{e3}$  is obtained for  $\alpha = \pi/2$ , which would mean that  $\theta_{12} = 0$ .

Since in seesaw models any symmetry of the neutrino mass matrix is supposed to manifest only at the seesaw scale, it is of interest to ask what radiative corrections would

do to this result. It is well known that radiative corrections are small in case of a normal hierarchy. For instance, using the approximative formulae from Ref. [12], we can estimate the  $\beta$ -function for the Dirac phase. Neglecting the smallest mass state  $m_1$  and using that  $|U_{e3}| \simeq \sqrt{\Delta m_\odot^2 / \Delta m_A^2}$ , we estimate that for the MSSM  $\dot{\delta} \sim 10^{-6} (1 + \tan^2 \beta)$ . In case of just the Standard Model (SM), we have  $\dot{\delta} \sim 10^{-6}$ . To put it another way, let us denote the parameter responsible for radiative corrections with  $\epsilon_1$ , where  $\epsilon_1 = c \frac{m_\tau^2}{16\pi^2 v^2} \ln \frac{M_X}{m_Z}$ , with  $c$  given by  $3/2$  in the SM and by  $-(1 + \tan^2 \beta)$  in the MSSM. As long as  $\epsilon_1 \ll 1$  (or  $\tan \beta \lesssim 50$ ) and  $\epsilon_1 \ll \epsilon$ , the predictions given above, in particular the one of maximal  $CP$  violation, remain unaffected.

The mass matrix from Eq. (4) could be modified by making the  $ee$  entry proportional to  $d\epsilon$ , a form which however could not be achieved by simple models based on  $U(1)$  charges. In this case the effective mass in neutrinoless double beta decay would be larger by a factor of  $1/\epsilon \sim \sqrt{\Delta m_A^2 / \Delta m_\odot^2}$  and the expression for solar neutrino mixing would be  $\tan 2\theta_{12} \simeq 2a \sqrt{1 + \cos \alpha} / (1 - d)$ . The expressions for  $J_{CP}$  and  $\Delta m_\odot^2$  would be more complicated, all other observables are however as above.

### 3 Inverted hierarchy

Now we break  $\mu$ - $\tau$  symmetry for the case of inverted hierarchy. In this case, the phase that breaks the symmetry could be located in various places. Here we consider the simplest case where the mass matrix is a special case of the broken flavor symmetry  $L_e - L_\mu - L_\tau$  [13, 14] and reads

$$\mathcal{M}_\nu = m_0 \begin{pmatrix} d\epsilon & a & a e^{i\alpha} \\ a & b\epsilon & f\epsilon \\ a e^{i\alpha} & f\epsilon & b\epsilon \end{pmatrix}, \quad (13)$$

where  $\epsilon \ll 1$  and  $a, b, d, f$  are real and of order one. Since obviously  $m_0 \simeq \sqrt{\Delta m_A^2}$  and the lower limit on the effective mass  $\langle m \rangle = m_0 d\epsilon$  is given by [7]  $\langle m \rangle \gtrsim \cos 2\theta_{12} \sqrt{\Delta m_A^2} \simeq 0.4 \sqrt{\Delta m_A^2}$ , we see that  $\epsilon$  should not be too small.

The Jarlskog invariant is proportional to

$$\text{Im} \{h_{12} h_{23} h_{31}\} \simeq 2\epsilon^2 a^4 b (f + (b + d) \cos \alpha) \sin \alpha + \mathcal{O}(\epsilon^3). \quad (14)$$

This and the exact expression vanish for  $\alpha = 0$  as in the case of normal hierarchy. Diagonalizing Eq. (13) gives (the procedure is described, e.g., again in Ref. [11])

$$\theta_{23} - \frac{\pi}{4} \sim \epsilon^2 \sin \alpha, \quad |U_{e3}| \simeq \frac{b}{\sqrt{2}a} \epsilon \sin \alpha \quad \text{and} \quad \sin \delta \simeq \cos \alpha. \quad (15)$$

Again, as in the case of normal hierarchy,  $\delta$  does formally not vanish for  $\alpha = 0$ . However, in this limit  $J_{CP}$  vanishes, which can be understood because  $U_{e3} = 0$  for  $\alpha = 0$ . Solar neutrino mixing is typically rather large, because it is in first order inversely proportional to  $\epsilon$ . A short approximate formula is possible when the entries in the lower  $\mu\tau$  block of Eq.

(13) are of order  $\epsilon^2$ , in which case  $|U_{e3}| \simeq \frac{b}{\sqrt{2}a} \epsilon^2 \sin \alpha$ , together with  $\theta_{23} - \frac{\pi}{4} \simeq \frac{bd}{\sqrt{2}a} \epsilon^3 \sin \alpha$  and  $\tan 2\theta_{12} \simeq \frac{2\sqrt{2}a}{d\epsilon}$ . For both possibilities, however, the strong dependence of  $\theta_{12}$  on the phase, which we encountered in case of a normal hierarchy, is lost. The large value of  $\theta_{12}$  reflects the well-known fact that the texture in Eq. (13) in the limit of  $\epsilon = 0$  produces maximal solar neutrino mixing. Sizable contributions from the charged lepton sector are then required to reach accordance with the data [14, 15]. For  $\theta_{12}^\nu = \pi/4$ , small  $|U_{e3}|^\nu \neq 0$  and for the most natural case of hierarchical charged lepton mixing the result is

$$\sin^2 \theta_{12} \simeq \frac{1}{2} - \frac{1}{\sqrt{2}} \sin \theta_{12}^\ell \quad \text{and} \quad |U_{e3}| \simeq \left| |U_{e3}|^\nu - \frac{\sin \theta_{12}^\ell}{\sqrt{2}} c_\alpha \right|, \quad (16)$$

plus higher order terms. The deviation from maximal solar neutrino mixing is connected to the magnitude of  $U_{e3}$ , where however cancellations can occur. We will give a more detailed analysis on charged lepton contributions below.

We can again analyze the case of the  $e\mu$  element being the complex conjugate of the  $e\tau$  element. Maximal  $CP$  violation in the sense of  $\delta = \pm\pi/2$  is predicted, as well as  $|U_{e3}| \simeq \epsilon \sqrt{2} b/a \cos \alpha \sin \alpha$  and  $\theta_{23} - \pi/4 \propto \epsilon^3 \sin \alpha$ . Again,  $\tan 2\theta_{12}$  is large because at first order it is inversely proportional to  $\epsilon$ .

As in the case of a normal hierarchy, effects of radiative corrections are rather small. Here the reason is that the pseudo-Dirac structure of the mass matrix (13) implies that the two heaviest mass states have basically opposite  $CP$  parities, which largely suppresses the running. Neglecting the smallest mass state and using  $|U_{e3}| \simeq \sqrt{\Delta m_\odot^2 / \Delta m_A^2}$ , one can estimate that  $\dot{\delta} \sim 10^{-5} (1 + \tan^2 \beta) \Delta m_A^2 / \Delta m_\odot^2 \sin \Delta\phi$ , where  $|\Delta\phi| \simeq \pi$  is the difference of the Majorana phases of the two leading mass states.

## 4 Contributions from the Charged Lepton Sector

We saw above that for the inverted hierarchy case and if we are too close to the  $L_e - L_\mu - L_\tau$  symmetry limit, one might require contributions from the charged lepton sector to fit observations. Here we analyze this possibility in more detail in order to investigate whether there are correlations between the  $CP$ -even and -odd observables for this case as well. Recall that the charged leptons strongly break  $\mu$ - $\tau$  symmetry anyway and therefore the case when the neutrino sector is  $\mu$ - $\tau$  symmetric and corrections stem from the charged lepton sector is surely appealing.

The basic idea is that the matrix  $U_\ell$  associated with the diagonalization of the charged leptons corrects a given neutrino mixing scheme. This could be the bimaximal [14, 15, 16] or the tribimaximal [17, 18] mixing scenario or, as in our case, the  $\mu$ - $\tau$  symmetric scenario [19]. In the last case, the neutrino mixing matrix (ignoring the Majorana phases) can be

written as:

$$\tilde{U}_\nu = \begin{pmatrix} \cos \theta_{12}^\nu & \sin \theta_{12}^\nu & 0 \\ -\frac{\sin \theta_{12}^\nu}{\sqrt{2}} & \frac{\cos \theta_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\sin \theta_{12}^\nu}{\sqrt{2}} & -\frac{\cos \theta_{12}^\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (17)$$

The angle  $\theta_{12}$  is a priori undefined, for instance, it was quite large for the matrix (13) and defined by  $\tan 2\theta_{12} = 2\sqrt{2}a/(1 - d\epsilon)$  for Eq. (4) and  $CP$  conservation [5]. Note that the following discussion is independent on the neutrino mass spectrum.

As has been shown in Ref. [14], one can in general express the PMNS matrix as

$$U = \tilde{U}_\ell^\dagger P_\nu \tilde{U}_\nu Q_\nu. \quad (18)$$

It consists of two diagonal phase matrices  $P_\nu = \text{diag}(1, e^{i\phi}, e^{i\omega})$  and  $Q_\nu = \text{diag}(1, e^{i\rho}, e^{i\sigma})$ , as well as two ‘‘CKM-like’’ matrices  $\tilde{U}_\ell$  and  $\tilde{U}_\nu$  which contain three mixing angles and one phase each, and are parametrized in analogy to Eq. (1). Out of the six phases present in Eq. (18), five stem from the neutrino sector. We denote the phase in  $\tilde{U}_\ell$  with  $\psi$ . The six phases will in general contribute in a complicated manner to the three observable ones. In case of one angle in  $U_\nu$  being zero (a result of  $\mu$ - $\tau$  symmetry), the phase in  $\tilde{U}_\nu$  is unphysical. Note that the 2 phases in  $Q_\nu$  do not appear in observables describing neutrino oscillations [14]. Let us assume that the matrix  $U_\ell$  contains only small angles, whose sines we denote by  $\lambda_{ij} \equiv \sin \theta_{ij}^\ell$ , with  $ij = 12, 13, 23$ . Keeping only the first order terms in the  $\lambda_{ij}$ , we get<sup>4</sup>

$$\begin{aligned} \sin^2 \theta_{12} &\simeq \sin^2 \theta_{12}^\nu - \frac{1}{\sqrt{2}} c_\phi \sin 2\theta_{12}^\nu \lambda_{12} + \frac{1}{\sqrt{2}} c_{\omega-\psi} \sin 2\theta_{12}^\nu \lambda_{13}, \\ |U_{e3}| &\simeq \frac{1}{\sqrt{2}} |\lambda_{12} + \lambda_{13} e^{i(\omega-\psi-\phi)}|, \\ \sin^2 \theta_{23} &\simeq \frac{1}{2} - \lambda_{23} c_{\omega-\phi}, \end{aligned} \quad (19)$$

where we introduced the notations  $c_\phi = \cos \phi$ ,  $c_{\omega-\psi} = \cos(\omega-\psi)$  and so on. The parameter  $\lambda_{23}$  does in first order not appear in  $\sin^2 \theta_{12}$  and  $|U_{e3}|$ , just as  $\lambda_{12,13}$  does not in  $\sin^2 \theta_{23}$ . We see from the expressions that — unless there are cancellations —  $|U_{e3}|$  is lifted from its value zero, and that the same small parameters  $\lambda_{12,13}$  appear in leading order also in the deviation from the original value of  $\theta_{12}$ . Leptonic  $CP$  violation is described by

$$J_{CP} \simeq \frac{1}{4\sqrt{2}} \sin 2\theta_{12}^\nu (\lambda_{12} s_\phi + \lambda_{13} s_{\omega-\psi}). \quad (20)$$

Consider now the case of a strong hierarchy in the charged lepton mixing, say, a ‘‘CKM-like’’ structure in the form of  $\lambda_{12} = \lambda$ ,  $\lambda_{23} = A\lambda^2$  and  $\lambda_{13} = B\lambda^3$  with  $A, B$  real and of order one. Then we have

$$\begin{aligned} \sin^2 \theta_{12} &\simeq \sin^2 \theta_{12}^\nu - \frac{1}{\sqrt{2}} c_\phi \sin 2\theta_{12}^\nu \lambda + \frac{1}{4} (2(c_{2\phi} + s_{2\phi}) \sin^2 \theta_{12}^\nu + 3 \cos 2\theta_{12}^\nu) \lambda^2, \\ |U_{e3}| &\simeq \frac{1}{\sqrt{2}} \lambda, \text{ where } J_{CP} \simeq \frac{\lambda}{4\sqrt{2}} \sin 2\theta_{12}^\nu s_\phi, \\ \sin^2 \theta_{23} &\simeq \frac{1}{2} + \frac{1}{4} (c_{2\phi} + s_{2\phi} - 4Bc_{\omega-\phi} - 2) \lambda^2, \end{aligned} \quad (21)$$

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<sup>4</sup>Setting in these equations  $\theta_{12}^\nu$  to  $\pi/4$  (to  $\sin^{-1} \sqrt{1/3}$ ) reproduces the formulae from [14] ([18]).



plus terms of  $\mathcal{O}(\lambda^3)$ . We see that the leading contribution to  $\sin^2 \theta_{12}$  depends on the same phase  $\phi$  to which leptonic  $CP$  violation is proportional. Since it follows that at leading order

$$\sin^2 \theta_{12} \simeq \sin^2 \theta_{12}^\nu - \sin 2\theta_{12}^\nu |U_{e3}| c_\phi, \quad (22)$$

there are some constraints on the possible values of  $\phi$ ,  $|U_{e3}|$  and  $\theta_{12}^\nu$ . Fixing  $\lambda = 0.22$  (which can be motivated by ideas such as Quark–Lepton–Complementarity [15]), we give in Fig. 2 the implied correlations between the parameters. We see that  $|U_{e3}| \simeq \lambda/\sqrt{2} \simeq 0.16$  and that atmospheric neutrino mixing can deviate sizably from maximal. The parameter  $\theta_{12}^\nu$ , which fixes the relative size of the parameters in a  $\mu$ – $\tau$  symmetric mass matrix, has to lie somewhere between  $\pi/6$  and  $3\pi/4$ . As can be seen, large  $CP$  violation is possible in these cases.

## 5 Summary

In summary, we have considered the possibility that leptonic  $CP$  violation owes its origin to the breaking of a  $\mu$ – $\tau$  symmetry, which in turn is motivated by the near maximal  $\theta_{23}$  and the near zero  $\theta_{13}$ . One consequence of this hypothesis is that interesting correlations between the size of  $U_{e3}$ , the magnitude of  $CP$  violation and even the solar neutrino mixing angle occur if the neutrino mass hierarchy is normal. Radiative corrections to these conclusions are mild and the results are easily testable in future neutrino experiments. For the case of inverted hierarchy there is no correlation between the solar neutrino mixing and the  $CP$  phase. We furthermore studied the possibility that the  $\mu$ – $\tau$  breaking originates in the charged lepton sector with the neutrino sector being  $\mu$ – $\tau$  symmetric and presented its consequences. Interestingly, also in this case there can be significant correlations between  $CP$ –even and –odd observables.

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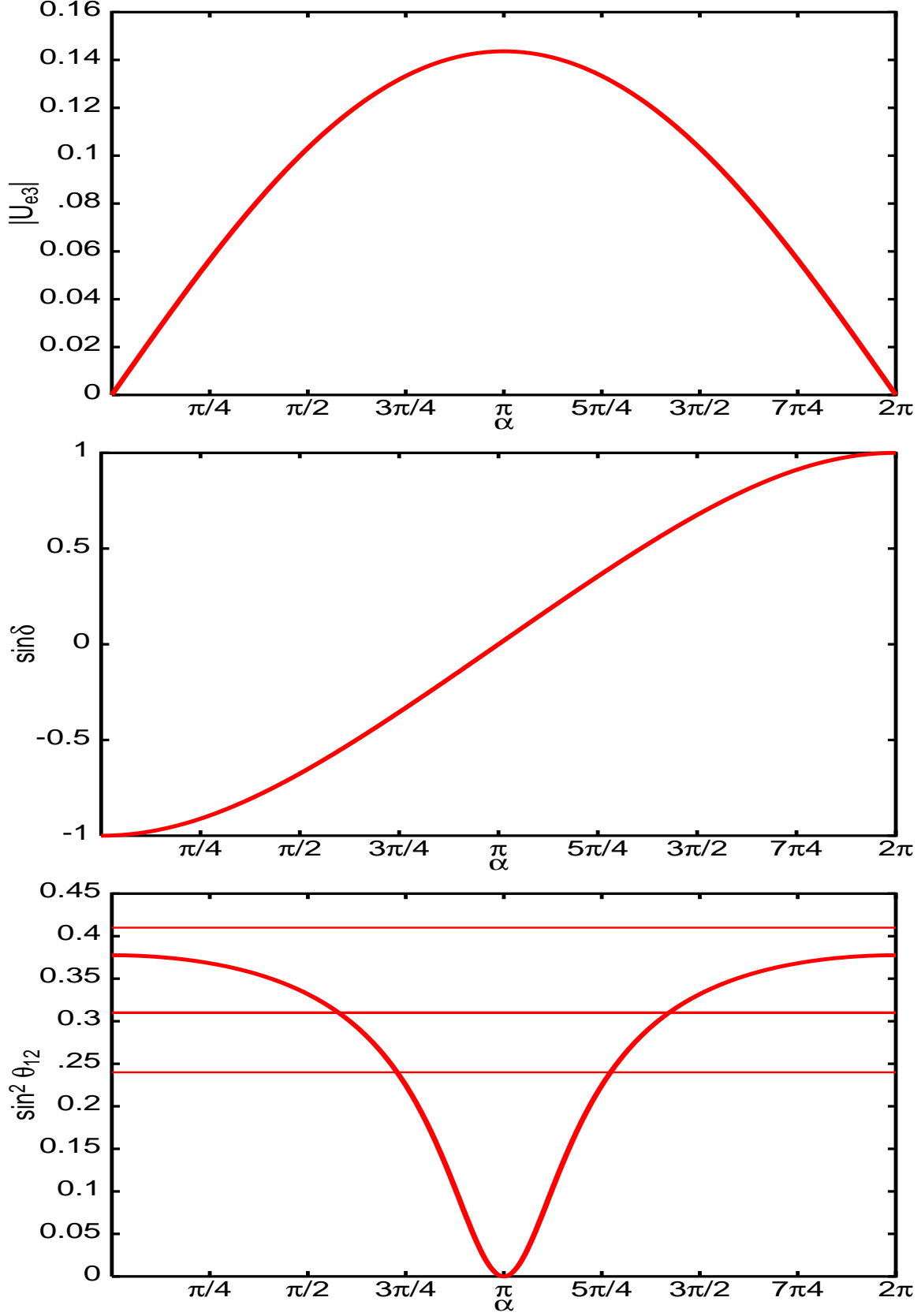


Figure 1: Behavior of  $|U_{e3}|$ ,  $\sin \delta$  and  $\sin^2 \theta_{12}$  as a function of the  $\mu$ - $\tau$  symmetry breaking phase  $\alpha$ , as it follows from Eq. (4). For this particular example we chose  $a = 1.4$  and  $\epsilon = 0.16$ . We also indicated the current best-fit value and  $3\sigma$  range of  $\sin^2 \theta_{12}$  from [10].

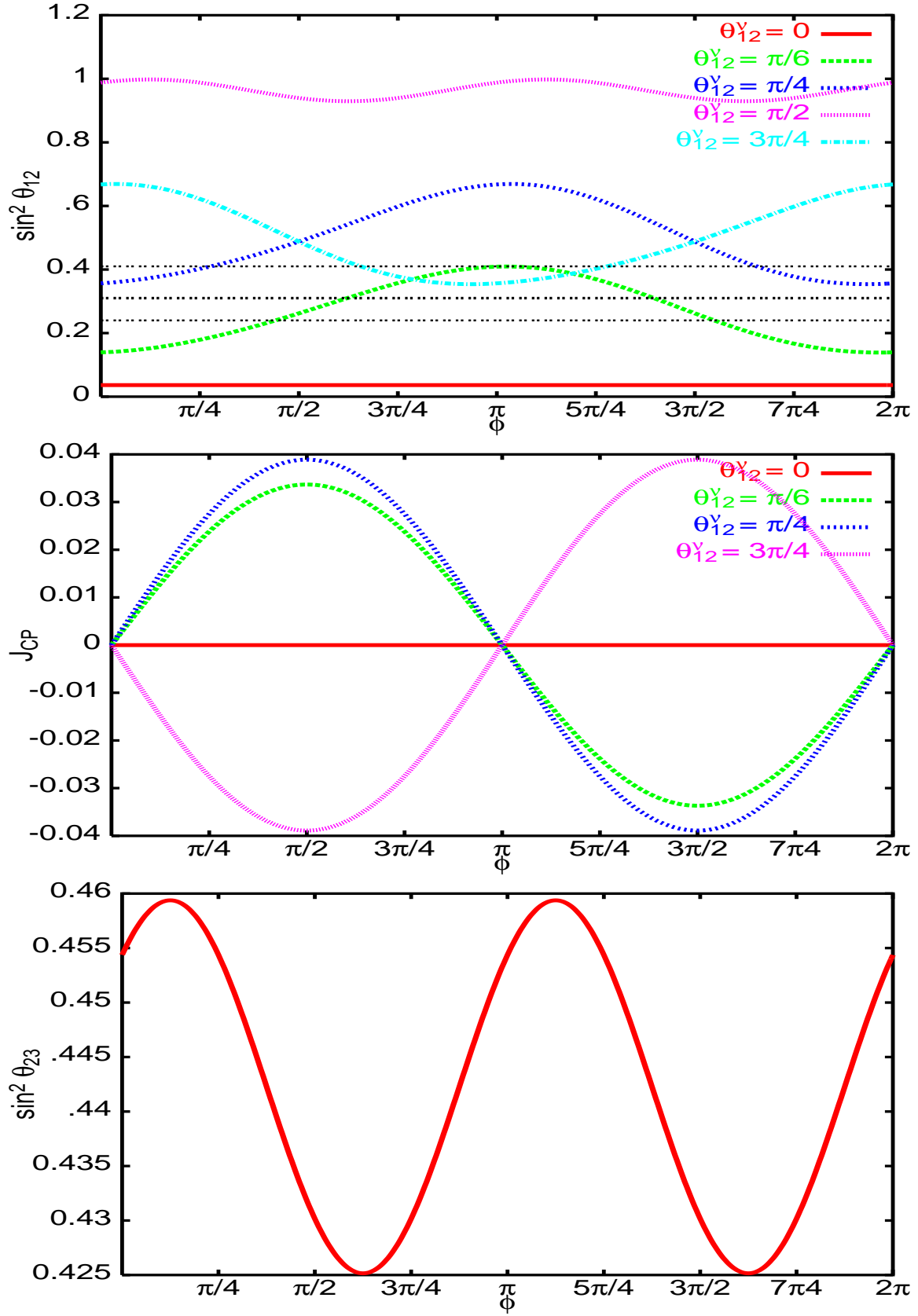


Figure 2: Behavior of  $\sin^2 \theta_{12}$  and  $J_{CP}$  as a function of the phase  $\phi$ , as it follows from Eq. (21). In this example  $|U_{e3}| \simeq 0.16$  and  $\psi = 60^\circ$ ,  $B = 0.8$  and  $\omega = 90^\circ$ .